Subspace coverings with multiplicities
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Abstract

We study at the problem of determining the minimum number $f(n, k, d)$ of affine subspaces of codimension d that are required to cover all points of $\mathbb{F}_2^n \setminus \{0\}$ at least k times while covering the origin at most $k - 1$ times. The case $k = 1$ is a classic result of Jamison, which was independently obtained by Brouwer and Schrijver for the case of hyperplane coverings, i.e., $d = 1$. The value of $f(n, 1, 1)$ also follows from a well-known theorem of Alon and Füredi about hyperplane coverings of finite grids in affine spaces over arbitrary fields.

Here we determine the value of this function exactly in various ranges of the parameters. In particular, we prove that for $k \geq 2^{n-d-1}$ we have $f(n, k, d) = 2^d k - \lfloor \frac{k}{2^{n-d}} \rfloor$, while for $n > 2^{2^d k - k - d + 1}$ we have $f(n, k, d) = n + 2^d k - d - 2$, and also study the transition between these two ranges. While previous work in this direction has primarily employed the polynomial method, we prove our results through more direct combinatorial and probabilistic arguments, and also exploit a connection to coding theory.