This exam consists of 8 problems.
The maximum number of points for this exam is 61.
At each question is indicated how many points it is worth.

Write down a solution for every question. Giving only the final answer will result in zero points.
Formulas

Trigonometry

\[ \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \]
\[ \cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \]
\[ \cos(2\alpha) = 2\cos^2(\alpha) - 1 \]
\[ \cos(2\alpha) = 1 - 2\sin^2(\alpha) \]
1. Consider the function \( f(x) = 3x^2 - x^3 \). The graph of \( f \) has an inflection point. The graph of \( f \) and its inflectional tangent \( l \) are drawn in Figure 1.

![Graph of f and tangent l](image)

**Figuur 1**

5p Find an equation of the inflectional tangent \( l \).

2. Consider the function \( f(x) = \frac{|2x - 1|}{3x + 5} + 1 \).

5p Find the equations of the asymptotes of the graph of \( f \).

CONTINUE WITH PROBLEM 3
3. The motion of a point $P$ is given by the equations of motion:

\[
\begin{align*}
    x(t) &= t^2 - 1 \\
y(t) &= t(t + 2)
\end{align*}
\]

The path of $P$ is drawn in Figure 2.

4p Calculate the exact orbital velocity with which $P$ crosses the positive $y$-axis.

\[\text{CONTINUE WITH PROBLEM 4}\]
4. The function $f$ with domain $(-\infty, 1]$ is given by $f(x) = \sqrt{1-x}$.

The plane region $V$ is enclosed by the graph of $f$, the $x$-axis and the $y$-axis. See Figure 3.

$$\begin{array}{c}
\begin{array}{c}
\text{Figuur 3}
\end{array}
\end{array}$$

4p (a) Calculate the exact area of $V$.

Revolving the plane region $V$ around the $y$-axis results in a body $L$.

4p (b) Calculate the exact volume of $L$.

The function $g$ is given by $g(x) = 1 - x$.

The area enclosed by the graphs of $f$ and $g$ is highlighted in grey in Figure 4.

The body $M$ results from revolving the grey area around the $x$-axis.

5p (c) Calculate the exact volume of $M$.

$$\begin{array}{c}
\begin{array}{c}
\text{Figuur 4}
\end{array}
\end{array}$$

CONTINUE WITH PROBLEM 5
5. Consider the function \( f(x) = 6 \sin(x) - \cos(2x) \).

The graph of \( f \) has infinitely many peaks. Figure 5 shows two periods of the graph of \( f \).

![Graph of f(x)](image)

**Figuur 5**

6p Calculate the exact \( x \)-coordinates of all peaks of the graph of \( f \).
6. Consider the circle $c$ with equation $c: (x - 14)^2 + (y - 8)^2 = 100$.

The circle $c$ intersects the $x$-axis in the points $A$ and $B$, see Figure 6. The line $k$ touches the circle $c$ in the point $A(8, 0)$.

(a) Find an equation of $k$.

It holds that $AB = \frac{3}{2} \cdot OA$.

(b) Prove this.

The line $l$ is perpendicular to the line segment $OM$ and passes through the midpoint of line segment $OM$.

(c) Find a vector representation of $l$.

The line $m: y = 2x$ intersects the circle $c$ in the points $C$ and $D$. In Figure 7, the situation of Figure 6 has been expanded with line $m$.

(d) Calculate the coordinates of $C$ and $D$.

CONTINUE WITH PROBLEM 7
7. Gegeven zijn de functies $f(x) = e^{x-1}$ en $g(x) = 4 - e^{1-x}$.

Consider the functions $f(x) = e^{x-1}$ and $g(x) = 4 - e^{1-x}$.

The line $x = p$ is a vertical line in between the intersection points of $f$ and $g$. This line $x = p$ intersects the graph of $f$ in point $A$ and the graph of $g$ in point $B$. See Figure 8 below.

There is one value of $p$ for which the length of $AB$ is maximal.

6p Calculate the exact maximum length of $AB$. 

THE FINAL QUESTION OF THIS EXAM IS ON THE NEXT PAGE
8. For every value of \( p > 0 \) a function \( f_p \) is defined by \( f_p(x) = \frac{p}{\sin(x)} \).

The graph of \( f_p \) has three extreme \( A, B \) and \( C \) on the interval \([0, 3\pi]\). The points \( A, B \) and \( C \) are the vertices of a triangle. The graph of \( f_p \) and the triangle \( ABC \) are drawn in Figure 9 for a certain value of \( p \).

\[ \text{Figuur 9} \]

8p Prove there is one value of \( p \) for which triangle \( ABC \) is an isosceles right-angled triangle. *Isosceles means that the triangle has two sides of equal length.*