Exam

Winter course Mathematics B

April 9th, 2020
10:00-12:30

This exam consists of 8 problems.
The maximum number of points for this exam is 61.
At each question is indicated how many points it is worth.

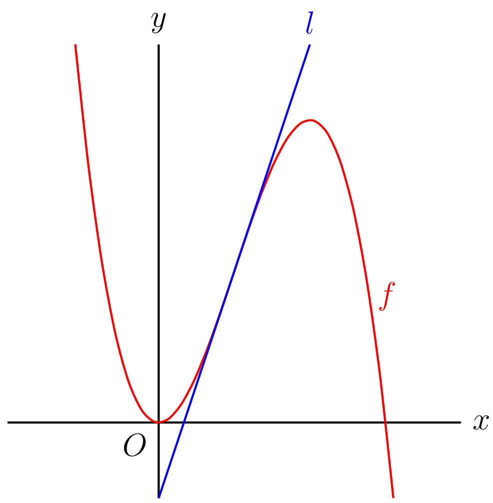
Write down a solution for every question. Giving only the final answer will result in zero points.

Formulas

Trigonometry

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$
$$\cos(2\alpha) = 2\cos^2(\alpha) - 1$$
$$\cos(2\alpha) = 1 - 2\sin^2(\alpha)$$

1. Consider the function $f(x) = 3x^2 - x^3$. The graph of f has an inflection point. The graph of f and its inflectional tangent l are drawn in Figure 1.



Figuur 1

5p Find an equation of the inflectional tangent l.

2. Consider the function $f(x) = \dfrac{|2x-1|}{3x+5} + 1$.

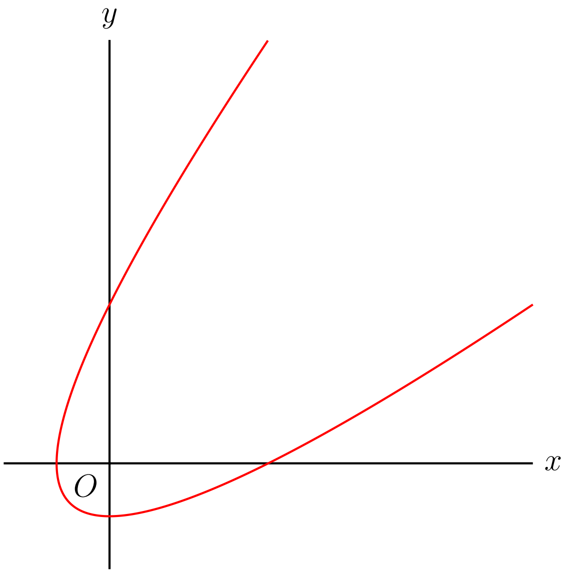
5p Find the equations of the asymptotes of the graph of f.

CONTINUE WITH PROBLEM 3

3. The motion of a point P is given by the equations of motion:

$$\begin{cases} x(t) = t^2 - 1 \\ y(t) = t(t+2) \end{cases}$$

The path of P is drawn in Figure 2.

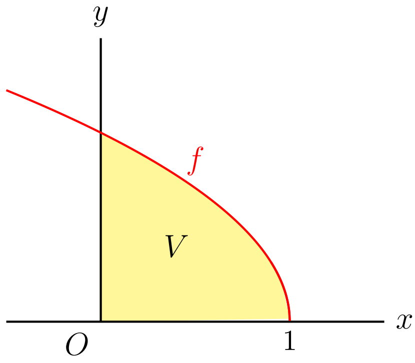


Figuur 2

4p Calculate the exact orbital velocity with which P crosses the positive y-axis.

CONTINUE WITH PROBLEM 4

4. The function f with domain $(-\infty, 1]$ is given by $f(x) = \sqrt{1-x}$.
The plane region V is enclosed by the graph of f, the x-axis and the y-axis. See Figure 3.



Figuur 3

4p (a) Calculate the exact area of V.

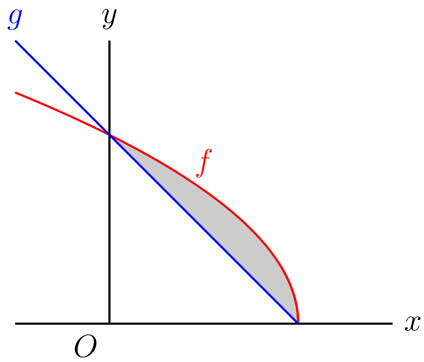
Revolving the plane region V around the y-axis results in a body L.

4p (b) Calculate the exact volume of L.

The function g is given by $g(x) = 1 - x$.
The area enclosed by the graphs of f and g is highlighted in grey In Figure 4.

The body M results from revolving the grey area around the x-axis.

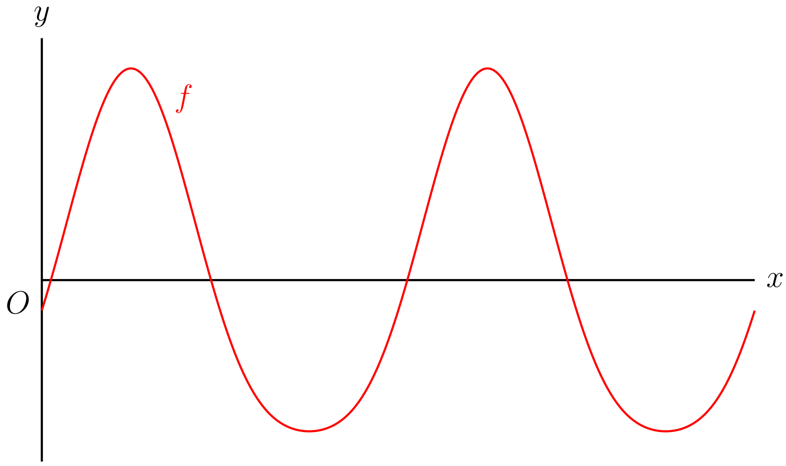
5p (c) Calculate the exact volume of M.



Figuur 4

CONTINUE WITH PROBLEM 5

5. Consider the function $f(x) = 6\sin(x) - \cos(2x)$.
The graph of f has infinitely many peaks. Figure 5 shows two periods of the graph of f.



Figuur 5

6p Calculate the exact x-coordinates of all peaks of the graph of f.

CONTINUE WITH PROBLEM 6

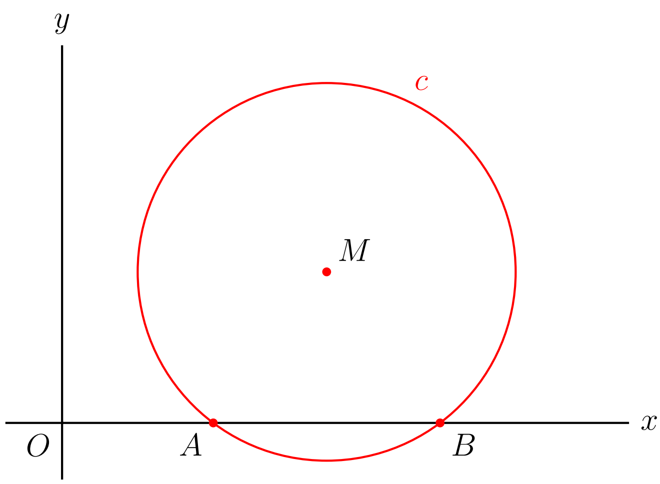
6. Consider the circle c with equation $c\colon (x-14)^2+(y-8)^2=100$.

The circle c intersects the x-axis in the points A and B, see Figure 6.
The line k touches the circle c in the point $A(8,0)$.

4p (a) Find an equation of k.

It holds that $AB = \frac{3}{2} \cdot OA$.

3p (b) Prove this.



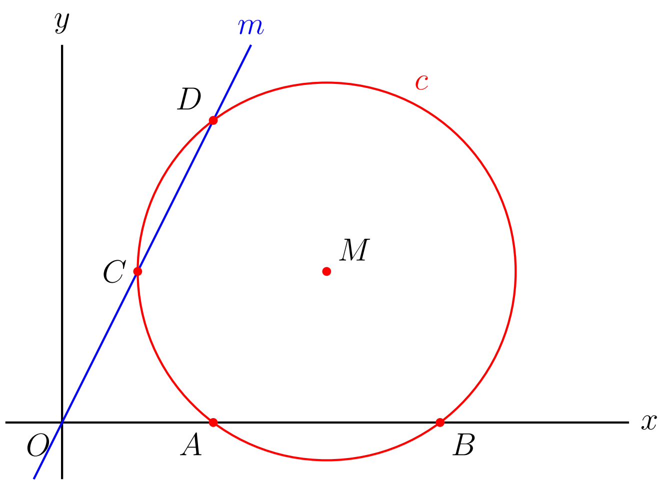
Figuur 6

The line l is perpendicular to the line segment OM and passes through the midpoint of line segment OM.

3p (c) Find a vector representation of l.

The line $m\colon y = 2x$ intersects the circle c in the points C and D. In Figure 7, the situation of Figure 6 has been expanded with line m.

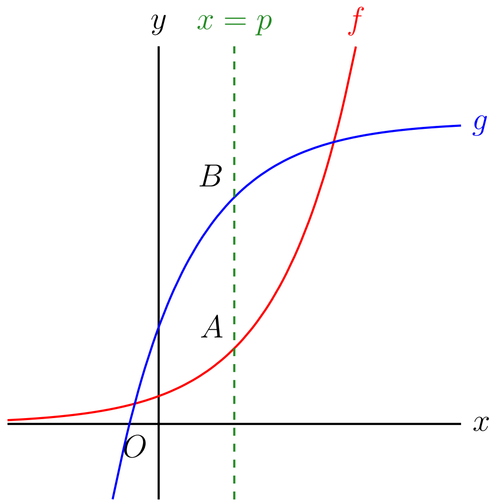
4p (d) Calculate the coordinates of C and D.



Figuur 7

CONTINUE WITH PROBLEM 7

7. Gegeven zijn de functies $f(x) = e^{x-1}$ en $g(x) = 4 - e^{1-x}$.
Consider the functions $f(x) = e^{x-1}$ and $g(x) = 4 - e^{1-x}$.
The line $x = p$ is a vertical line in between the intersection points of f and g. This line $x = p$ intersects the graph of f in point A and the graph of g in point B. See Figure 8 below.



Figuur 8

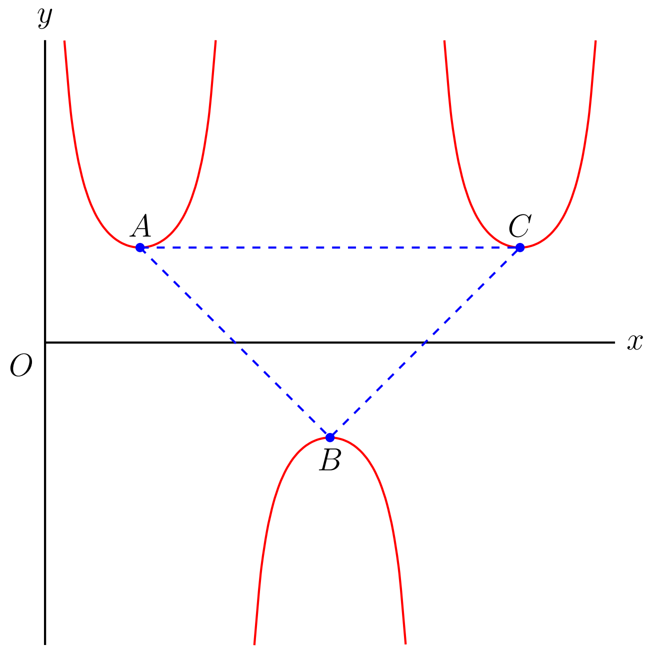
There is one value of p for which the length of AB is maximal.

6p Calculate the exact maximum length of AB.

THE FINAL QUESTION OF THIS EXAM IS ON THE NEXT PAGE

8. For every value of $p > 0$ a function f_p is defined by $f_p(x) = \dfrac{p}{\sin(x)}$.

The graph of f_p has three extreme A, B and C on the interval $[0, 3\pi]$. The points A, B and C are the vertices of a triangle. The graph of f_p and the triangle ABC are drawn in Figure 9 for a certain value of p.



Figuur 9

8p Prove there is one value of p for which triangle ABC is an isosceles right-angled triangle. *Isosceles means that the triangle has two sides of equal length.*