UNIVERSITY OF AMSTERDAM
FACULTY FNWI Practice exam mathematics A; 2.5 hour

This is a 2.5 hour exam.
The exam consists of 11 questions.
A maximum of 72 marks can be obtained for this exam.
In front of each item is stated how many marks it is worth.

There is a formula chart at the beginning of the exam. Other formula charts are not allowed. It is not permitted to use a calculator.

Write down a full algebraic solution at each question. No marks will be given for an answer without a worked-out solution.

Good luck!

Formulas

Differentiation

name of rule	**form of formula**	**derivative**
sum rule	$f + g$	$(f + g)' = f' + g'$
difference rule	$f - g$	$(f - g)' = f' - g'$
product rule	$f \cdot g$	$(f\,g)' = f'g + f\,g'$
quotient rule	$\frac{f}{g}$	$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$
chain rule	$f(x) = h\,(u(x))$	$f'(x) = h'\,(u(x)) \cdot u'(x)$

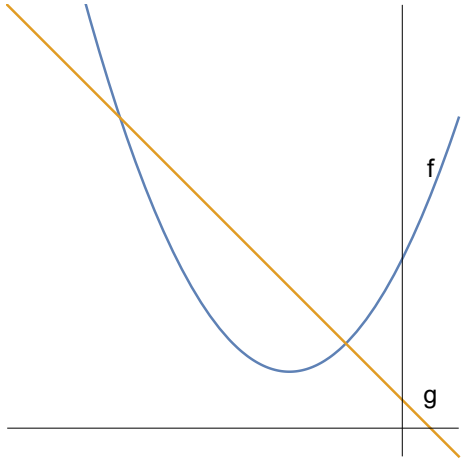
Logarithm

rule	**conditions**
$\log_g(a) + \log_g(b) = \log_g(ab)$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
$\log_g(a) - \log_g(b) = \log_g\left(\frac{a}{b}\right)$	$g > 0,\ g \neq 1,\ a > 0,\ b > 0$
$\log_g\left(a^k\right) = k \cdot \log_g(a)$	$g > 0,\ g \neq 1,\ a > 0$
$\log_c(a) = \frac{\log_g(a)}{\log_g(c)}$	$g > 0,\ g \neq 1,\ a > 0,\ c > 0,\ c \neq 1$

1. Two functions f and g are given by

$$f(x) = x^2 + 4x + 6 \quad \text{en} \quad g(x) = 1 - 2x.$$

The graphs of the functions f and g are drawn in Figure 1.



Figuur 1: The graphs of the functions f and g.

3m (a) Calculate the coordinates of the intersection points of f and g. Give an exact answer.

2m (b) Calculate the product of the functions f and g. In other words: determine the function $h(x)$ where $h(x) = f(x) \cdot g(x)$. Simplify the resulting polynomial as much as possible.

3m (c) Calculate the x-coordinates of the extreme values of the function $h(x) = f(x) \cdot g(x)$. Give an exact answer. It is *not* neccesary to point out which of the extreme values is a minimum or maximum.

3m (d) For which value of x between -5 and -1 is there a maximum in the vertical distance between the functions f and g?

2. Assume that the amount of duckweed in a pond doubles every three days. The surface area of the pond covered by duckweed on the first day is equal to 0.5 m^2.

1m (a) What is the growth factor g per day? Give an exact answer.

2m (b) What is the general formula which describes the covered surface of the pond by duckweed, with time unit t in days?

3m (c) How many days does it take before the duckweed covers an aera of 13 m^2 of the pond? Give an exact answer.

2m (d) After a while the nutrition in the water for the duckweed is not enough anymore. The duckweed dies off with 30% a week. What is the corresponding growth factor per two weeks? Give an exact answer.

3. Consider the two functions below. Determine the *zeros*, *minima*, and *maxima* of these functions. Furthermore, determine the values of x for which the functions are defined, the so-called *domain*, and which values the functions can take, the so-called *range*.

6m (a)
$$f(x) = -2x^3 + x^2 + 4x$$

5m (b)
$$g(x) = \frac{4x - 5}{10 - 3x}$$

4. Solve the following equations. Give an exact answer.

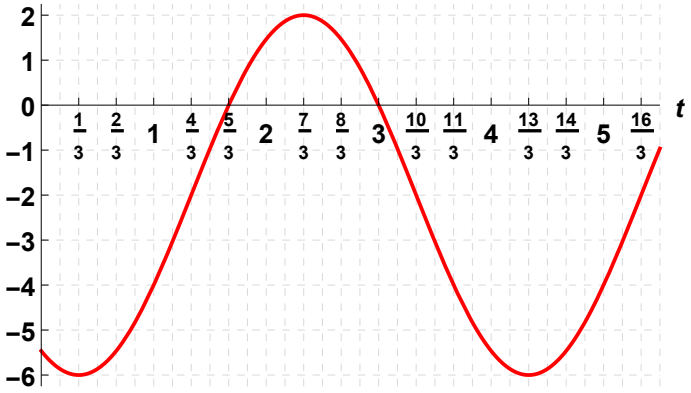
4m (a) $\sqrt{6x - 2} = 2x$

4m (b) $7^{1-x} = \sqrt{7}$

6m 5. The general formula for a sine wave is

$$f(t) = B + A\sin(2\pi ft + \phi)$$

Write down the function rule $f(t)$ of the sine wave of the following graph.



Figuur 2: The graph of the sine wave f.

6. Differentiate the following functions:

4m (a)
$$f(x) = \log_3(2x) - 4$$

2m (b)
$$g(x) = x\sqrt{x}$$

4m 7. Calculate the following sum. Use the sum formulas. Give an exact answer.
$$\sum_{k=3}^{8}(3 \cdot 2^k)$$

8. A basket contains 25 lemons and 5 limes. Someone randomly grabs five fruits from the basket.

3m (a) What is the probability that five limes are grabbed?

3m (b) What is the probability that more than three limes are grabbed?

Give exact answers.

2m 9. To decorate an edge of tile on a wall in a bathroom, with a length of 20 tiles, someone bought 20 tiles in five different colours (four tiles of each colour). How many different edges of tiles can be composed using the bought tiles? Tiles of the same colour may sit next to each other.

2m 10. At a four-choice test, you guess all twenty questions. Calculate the probability that you answered half of the questions correctly. Provision of the calculation method suffices; you do not have to calculate the integer value.

11. A manufacturer produces beer bottles with a mean volume of 330 mL and a standard deviation of 10 mL.

4m (a) What is the probability that a randomly selected beer bottle has a volume greater than 335 mL?

4m (b) What is the probability that the volume of a randomly selected beer bottle deviates more than 15 mL from the volume specified on the bottle?

Use the attached table of the cumulative standard normal distribution in this exercise. Round your final answer for each question to a precision of three decimals.

Cumulative standard normal distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Figuur 3: The table for the cumulative standard normal distribution. For example $\Phi(1.65) = P(Z \leq 1.65) = 0.9505$.